Assignment 1: Stagnation and Critical Properties

Compressible Flows - Master Course in Space and Astronautical Engineering

Sapienza University of Rome

Problem 1. *An aircraft is flying at a cruising speed of* $V_\infty = 250\,\mathrm{m/s}$ *at an altitude where the atmospheric pressure* p_∞ *is* $54.05\,\mathrm{kPa}$ *and the air temperature* T_∞ *is* $255.7\,\mathrm{K}$. *What are the stagnation temperature* $T_{0\infty}$ *and the stagnation pressure* $p_{0\infty}$*?*

Solution 1. *We assume that the flow is isentropic, and that air is an ideal gas with constant specific heats, such that:*

$$C_p = 1.005\,\mathrm{kJ/kg\,K} \quad \textit{and} \quad \gamma = 1.4\,. \tag{1}$$

For a steady, inviscid, and adiabatic flow, the total or stagnation enthalpy *is constant:*

$$h_{01} = h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2 = h_{02}\,, \tag{2}$$

and for an ideal gas, the enthalpy can be expressed as:

$$h = C_p\,T\,. \tag{3}$$

Thus, by introducing the so called total or stagnation temperature*:*

$$T_0 = T + \frac{V^2}{2C_p} \tag{4}$$

for an ideal gas, Eq. 2 can be rewritten as:

$$T_{01} = T_{02}\,. \tag{5}$$

As a results, it follows that the stagnation temperature is the temperature that an ideal gas reach when brought to rest though an adiabatic process. Indeed, if $V_2 = 0$*, one obtains that:*

$$T_{02} = T_2 = T_{01} = T_1 + \frac{V_1^2}{2C_p} \quad \Rightarrow \quad T_{02} = T_1 + \frac{V_1^2}{2C_p}\,, \tag{6}$$

In our case, we have that the stagnation temperature is:

$$T_{0\infty} = T_\infty + \frac{V_\infty^2}{2C_p} = 286.8\mathrm{K}\,. \tag{7}$$

On the other hand, pressure and temperature for an adiabatic process are related by the expression:

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}\,. \tag{8}$$

It follows that:

$$p_{0\infty} = p_\infty \left(\frac{T_{0\infty}}{T_\infty}\right)^{\frac{\gamma}{\gamma-1}} = 80.77\,\mathrm{kPa}\,. \tag{9}$$

If instead one wants to use the isentropic tables for the air, the infinite Mach number of the flow $M_\infty = V_\infty/a_\infty$ *is needed, and thus the speed of sound of the air* a_∞ *is needed. For an ideal gas, we have:*

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = \gamma R T\,, \tag{10}$$

where R *is the specific gas constant, which for the air is* $R = 287\,\mathrm{J/kg\,K}$. *For our case, we thus have that:*

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{V_\infty}{\sqrt{\gamma R T}} = 0.78 \tag{11}$$

At this point, we can find on the isentropic tables the ratios T/T_0 *and* p/p_0 *in correspondence of the Mach number found, and we can obtain the same results in Eqs. 7-9.*

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Problem 2. *Calculate the stagnation temperature and pressure for air, assumed as ideal gas, in a duct and at the following conditions:*

p [MPa]	T [K]	V [m/s]
0.25	380	400
0.15	380	400
0.15	480	400
0.15	380	500

Solution 2. *The results are reported in order.*

p_0 [MPa]	T_0 [K]
0.4864	459.6
0.2919	459.6
0.2566	559.6
0.4041	504.4

Problem 3. *In a device, the air flow has a stagnation pressure* $p_0 = 0.4\,\mathrm{MPa}$*, a stagnation temperature* $T_0 = 400\,^\circ\mathrm{C}$ *and a velocity* $V = 520\,\mathrm{m/s}$*. What are the static pressure and temperature, in* Pa *and* K *respectively? Assume a constant* $C_p = 1.005\,\mathrm{kJ/kg\,K}$*.*

Solution 3. $p = 1.833 \cdot 10^5\,\mathrm{Pa}$, $T = 538.6\,\mathrm{K}$

Problem 4. *A duct is fed by a reservoir with air at a pressure of* $200\,\mathrm{kPa}$ *and a temperature of* $150\,\mathrm{K}$*. In the section where the flow reaches the critical Mach number* $M^* = 2$*, evaluate the Mach number, the velocity and the temperature of the fluid.*

Solution 4. $M = 3.16$, $V = 448\,\mathrm{m/s}$, $T = 50.0\,\mathrm{K}$.

References

[1] YA Cengel and JM Cimbala. *Fluid Mechanics. Fundamentals and Applications.* New York: McGraw-Hill, 2018.

[2] Giorgio Graziani. *Aerodinamica.* Rome: Casa Editrice Università La Sapienza, 2007.