Assignment 2: One-Dimensional Isentropic Flows

Compressible Flows - Master Course in Space and Astronautical Engineering

Sapienza University of Rome

Problem 1. *Carbon dioxide ($C_p = 0.846\, kJ/kg\, K$, $\gamma = 1.289$, $R = 0.1889\, kJ/kg\, K$) flows steadily through a convergent-divergent duct at a mass flow rate $\dot{m} = 3.00 kg/s$. The carbon dioxide enters the duct at a pressure of $1400\, kPa$ and $200°C$ with a negligible velocity, and it expands in the nozzle. The duct is designed so that the flow can be approximated as isentropic. Determine the density, velocity, flow area, and Mach number at each location along the duct that corresponds to an overall pressure drop of $200\, kPa$.*

Solution 1. *Since the inlet velocity is negligible, we can approximate the stagnation pressure and temperature of the flow as the inlet quantities:*

$$p_0 = 1400\, kPa \quad T_0 = 200 + 273.15\, K \approx 473\, K\,. \tag{1}$$

The section that has a pressure drop $\Delta p = 200\, kPa$, has thus a pressure $p = 1400 - 200\, kPa = 1200\, kPa$. Since the flow is isentropic, we have that the temperature of the unknown section is:

$$T = T_0 \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 457\, K\,. \tag{2}$$

Given the conservation of the total temperature along the duct, we have that:

$$T_0 = T + \frac{V^2}{2C_p} \Rightarrow V = \sqrt{2C_p(T_0 - T)} \approx 164\, m/s\,. \tag{3}$$

From the ideal gas law:

$$\rho = \frac{p}{RT} = 13.9\, kg/m^3\,, \tag{4}$$

and thus from the expression of the mass flow rate, we obtain:

$$A = \frac{\dot{m}}{\rho V} = 1.31 \cdot 10^{-3}\, m^2\,. \tag{5}$$

Finally, the speed of sound is:

$$a = \sqrt{\gamma R T} = 333.6\, m/s\,, \tag{6}$$

and hence the Mach number of the section is:

$$Ma = \frac{V}{a} = 0.493\,. \tag{7}$$

Problem 2. *Given the data of the previous problem, evaluate what are the critical temperature and pressure of the flow.*

Solution 2. $T^* = 413\, K$, $p^* = 767\, kPa$.

Problem 3. *Air enters a converging nozzle at $207\, kPa$, $77°C$, and at a velocity of $137\, m/s$. Approximating the flow as isentropic, determine the pressure and temperature of air at the location where the air velocity equals the speed of sound. What is the exit-to-entrance area ratio?*

Solution 3. $p = 120\, kPa$, $T = 292\, K$, $A_e/A_{in} = 0.574$.

Problem 4. *Air enters a converging-diverging nozzle at a pressure $1.0\, MPa$ and a temperature $800\, K$ with negligible velocity. The flow is steady, one-dimensional, and isentropic with $\gamma = 1.4$. For an exit Mach number of $Ma = 2$ and a throat area $A_t = 20\, cm^2$, determine:*

- *the throat conditions;*
- *the exit plane conditions, including the exit area,*
- *the mass flow rate through the nozzle.*

Solution 4.

- $p_t = 0.5283\,MPa$, $T_t = 666.6\,K$, $\rho_t = 2.761\,kg/m^3$
- $p_e = 0.1278\,MPa$, $T_e = 444.5\,K$, $\rho_e = 1.002\,kg/m^3$, $A_e = 33.75\,cm^2$, $V_e = 845.1\,m/s$
- $\dot{m} = 2.86\,kg/s$

Problem 5. *Consider an isentropic flow through a converging duct fed by a tank at pressure* $200\,kPa$*. Evaluate the minimum pressure that one can reach in the duct.*

Solution 5. $p_{min} = 105.66\,kPa$

Problem 6. *A tank at pressure* $150\,kPa$ *feeds a converging-diverging duct with exit-to-throat area ratio* $A_e/A_t = 3$*. Evaluate the design back pressure and the value of the back pressure that is such to obtain a subsonic flow in the entire duct but with sonic throat.*

Solution 6. $p_{design} = 7.07\,kPa$, $p_{sub-sonic} = 146.1\,kPa$.

Problem 7. *Consider a converging-diverging fed by a tank at pressure* $150\,kPa$*, and operating at an ambient pressure* $9.79\,kPa$*. Evaluate the exit Mach number and the exit-to-throat area ration.*

Solution 7. $M = 2.43$, $A_e/A_t = 2.47$.

Problem 8. *Consider a converging channel with inlet section of* $0.5\,m^2$ *and an outlet section* $0.3\,m^2$*. Evaluate the exit Mach number when the inlet Mach number is* $M = 0.15$ *and* $M = 4.5$*.*

Solution 8. $M_e^{sub} = 0.26$, $M_e^{super} = 3.90$.

References

[1] YA Cengel and JM Cimbala. *Fluid Mechanics. Fundamentals and Applications.* New York: McGraw-Hill, 2018.

[2] Giorgio Graziani. *Aerodinamica.* Rome: Casa Editrice Università La Sapienza, 2007.