Assignment 3: de Laval Nozzles

Compressible Flows - Master Course in Space and Astronautical Engineering

Sapienza University of Rome

Problem 1. *Air ($\gamma = 1.4$) enters a converging-diverging nozzle with throat area $A_t = 20\,cm^2$ at a pressure $1.0\,MPa$ and a temperature $800\,K$ with negligible velocity. and, at design conditions, the exit Mach number is $Ma = 2$. If the air flowing through the nozzle experiences a normal shock wave at the exit plane, determine the following quantities after the shock:*

- *stagnation pressure, static pressure, stagnation temperature, static temperature;*
- *the exit velocity;*
- *the mass flow rate through the nozzle.*

Solution 1. *Since the velocity is negligible at the inlet, the pressure and the temperature given indicate the corresponding total quantities before the shock:*

$$p_{01} = 1.0\,MPa, \quad T_{01} = 800\,K\,. \tag{1}$$

Before evaluating the jump of the fluid properties across the final shock, we need to charaterise them completely before *the shock,* i.e. *at the exit section.*
For $M_e = 2$ at the exit section, we can read on the isentropic tables that:

$$M = 2 \quad \Rightarrow \quad \frac{A}{A^*} = 1.6875 \quad \frac{p}{p_{01}} = 0.1278 \quad \frac{T}{T_{01}} = 0.5556\,. \tag{2}$$

Thus, it follows that the conditions just before the exit section are:

$$A_1 = \frac{A}{A^*}A_t = 33.75\,cm^2 \quad p_1 = \frac{p}{p_{01}}p_{01} = 0.1278\,MPa \quad T_1 = \frac{T}{T_{01}}T_{01} = 444.5\,K \tag{3}$$

The exit conditions that we just determined represent the conditions before the normal shock wave. Thus, if we read the normal shock wave table in correspondence of the exit Mach number $M = 2$, we have:

$$M_2 = 0.5774 \quad \frac{p_{02}}{p_{01}} = 0.7209 \quad \frac{p_2}{p_1} = 4.5000 \quad \frac{T_2}{T_1} = 1.6875\,, \tag{4}$$

and thus:

$$p_{02} = \frac{p_{02}}{p_{01}}p_{01} = 0.721\,MPa \quad p_2 = \frac{p_2}{p_1}p_1 = 0.575\,MPa \quad T_2 = \frac{T_2}{T_1}T_1 = 750\,K\,. \tag{5}$$

What about the total temperature? Note that the conservation of energy requires the stagnation enthalpy to remain constant across the shock, and so that $T_{02} = T_{01} = 800\,K$!
In order to determine the exit velocity (after the shock), we now know the subsonic Mach number M_2 and we can derive the speed of sound after the shock, given T_2. Thus:

$$V_2 = M_2 a_2 = M_2\sqrt{\gamma R T_2} = 317\,m/s. \tag{6}$$

Finally, given the velocity and the ideal gas law, the mass flow rate is:

$$\dot{m} = \rho_2 V_2 A_e = \frac{p_2}{RT_2}V_2 A_e = 2.86\,kg/s\,. \tag{7}$$

Problem 2. *Air ($Cp = 1.005 kJ/kg\,K$) flows in a de Laval nozzle fed by a tank in which the air is still and at a temperature of $290\,K$. What is the maximum theoretical velocity that the flow can reach?*

Solution 2. $V = 763\,m/s$

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Problem 3. *Consider a converging-diverging nozzle fed by a tank containing air ($R = 287\,K/kg\,K$, $\gamma = 1.4$). Stagnation pressure and temperature inside the tank are $p_0 = 3.5\,MPa$ and $T_0 = 2500\,K$ and the throat area is $A_t = 0.35\,m^2$. Considering that, under design conditions, the Mach number at the exit section is $M_e = 3.5$. Evaluate:*

- *the value of the exit area;*
- *the static pressure, temperature and density at the exit section;*
- *the overall mass flow rate throught the nozzle;*
- *the static pressures characterising the nozzle functioning (limit subsonic, design, and overexpanded).*

Solution 3.

- $A_e = 2.376\,m^2$
- $p_e = 0.0459\,MPa$, $T_e = 724.65\,K$, $\rho_e = 0.221\,kg/m^3$
- $\dot{m} = 990\,kg/s$
- $p_{sub} = 3.48\,MPa$, $p_{des} = 0.0459\,MPa$, $p_{over} = 0.648\,MPa$

Problem 4. *Consider the same geometry and inlet conditions of the previous problem. Assuming that a normal shock takes place at the section with area $A_s = 0.40\,m^2$:*

- *draw a diagram of total pressure and temperature along the nozzle;*
- *evaluate the static pressure before and after the shock;*
- *evaluate the Mach number before and after the shock;*
- *evaluate the static pressure and the Mach number at the exit.*

Solution 4.

- *The total pressure is constant before the shock, has a sudden drop in correspondence of the shock and then is constant again until the exit but with a lower value. The total temperature is constant*
- $p_1 = 1.02\,MPa$, $p_2 = 2.34\,MPa$
- $M_1 = 1.45$, $M_2 = 0.72$
- $p_e = 3.29\,MPa$, $M_e = 0.09$

Problem 5. *In the divergent of a de Laval nozzle in which air flows, a normal shock wave takes place where the Mach number is equat to 2. Evaluate the Mach number after the shock and the pressure ratio across the shock.*

Solution 5. $M = 0.577$, $p_2/p_1 = 4.5$.

Problem 6. *A de Laval nozzle of exit-to-throat area ratio $A_e/A_t = 2.5$ is fed by a tank with air at total pressure $p_0 = 120\,kPa$. Evaluate the back pressure such that a normal shock of maximum intensity takes place at the exit section, and the corresponding exit Mach number.*

Solution 6. $p = 52.3\,kPa$, $M = 0.52$.

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Problem 7. *A tank with air at total pressure p_0 feeds a de Laval nozzle with exit-to-throat area ratio $A_e/A_t = 2$. Evaluate the exit Mach number if a shock takes place at the section such that $A_s/A_t = 1.5$.*

Solution 7. $M = 0.4$

References

[1] YA Cengel and JM Cimbala. *Fluid Mechanics. Fundamentals and Applications.* New York: McGraw-Hill, 2018.

[2] Giorgio Graziani. *Aerodinamica.* Rome: Casa Editrice Università La Sapienza, 2007.