Assignment 3+: Shocks and Nozzles

Compressible Flows - Master Course in Space and Astronautical Engineering

Sapienza University of Rome

Problem 1. *Air enters a normal shock at* $26\,kPa$, $230\,K$, *and* $815\,m/s$. *Calculate the stagnation pressure and Mach number upstream of the shock, as well as pressure, temperature, velocity, Mach number, and stagnation pressure downstream of the shock.*

Solution 1. $p_{01} = 587\,kPa$, $p_{02} = 253\,kPa$, $M_1 = 2.68$, $M_2 = 0.50$, $p_2 = 214\,kPa$, $T_2 = 534\,K$, $v_2 = 230\,m/s$.

Problem 2. *A conic divergent nozzle is fed by a reservoir with air* ($R = 297\,J/kgK$, $\gamma = 1.4$) *at* $7 \cdot 10^5\,Pa$ *and* $500\,K$. *The design ambient pressure is* $0.26 \cdot 10^4\,Pa$, *the semi-opening angle* δ *is equal to* $15°$ *and the mass flow rate under design conditions is* $9.54\,kg/s$. *What is the length of the nozzle?*

Solution 2. $L = 0.548\,m$.

Problem 3. *A conic nozzle has a maximum area of* $4\,m^2$ *and a minimum area of* $2\,m^2$. *A supersonic flow with Mach number* M *equal to* 4 *feeds the nozzle, in the first case from the maximum area, and in the second case from the minimum area. What are the Mach numbers in correspondence of the relative outlet openings in the two cases?*

Solution 3.
Flow from maximum opening (convergent nozzle): $M_{out} = 3.25$.
Flow from minimum opening (divergent nozzle): $M_{out} = 4.80$.

Problem 4. *Air enters a converging-diverging nozzle at* $1.2\,MPa$ *with a negligible velocity. Approximating the flow as isentropic, determine the back pressure that would result in an exit Mach number of* 1.8.

Solution 4. $p = 209\,kPa$.

Problem 5. *Air enters a nozzle at* $0.5\,MPa$, $420\,K$, *and a velocity of* $110\,m/s$. *Approximating the flow as isentropic, determine the pressure and temperature of air at a location where the air velocity equals the speed of sound. What is the ratio of the area at this location to the entrance area?*

Solution 5. $p = 278\,kPa$, $T = 355\,K$, $A^*/A_e = 0.45$.

Problem 6. *Air enters a converging-diverging nozzle with low velocity at* $2.4\,MPa$ *and* $120°C$. *If the exit area of the nozzle is* 3.5 *times the throat area, what must the back pressure be to produce a normal shock at the exit plane of the nozzle?*

Solution 6. $p = 0.794\,MPa$.

Problem 7. *What must the back pressure be in the previous exercise for a normal shock to occur at a location where the cross-sectional area is twice the throat area?*

Solution 7. $p = 1.43\,MPa$.

Problem 8. *Air enters a converging-diverging nozzle at* $1\,MPa$ *and* $300\,K$ *with a low velocity. If a normal shock wave occurs at the exit plane of the nozzle at* $M = 2.4$, *determine the pressure, temperature, Mach number, velocity, and stagnation pressure after the shock wave.*

Solution 8. $p_2 = 448\,kPa$, $T_2 = 284\,K$, $M_2 = 0.523$, $v_2 = 177\,m/s$, $p_{02} = 540\,kPa$.

References

[1] YA Cengel and JM Cimbala. *Fluid Mechanics. Fundamentals and Applications.* McGraw-Hill, 2018.

[2] Giorgio Graziani. *Aerodinamica.* Casa Editrice Università La Sapienza, 2007.

[3] Renato Paciorri. *Esercizi di Gasdinamica.* Edizioni Ingegneria 2000, 2000.