Assignment 4: Small Perturbations

Compressible Flows - Master Course in Space and Astronautical Engineering

Sapienza University of Rome

Problem 1. *A supersonic flow with* $M_\infty = 1.25$ *encounters the bump in Figure 1a, with* $\delta = 2^\circ$*. Determine the pressure coefficient on each of the numbered surfaces, according to the small perturbation theory.*

Solution 1. *According to the linear small perturbation theory, the solution can be determined by superposition of the effects and the variation of the pressure coefficient induced by a flow deviation of angle* θ *is given by:*

$$\Delta C_p = \frac{2\theta}{\sqrt{M_1^2 - 1}} \tag{1}$$

In the particular case under study, a compression wave originates from the feet of the first ramp since the flow must respect the boundary condition imposed by the angle δ *of the ramp itself. For the same principle, a rarefaction wave originates from the top of the bump in order to accomodate the downward flow deviation of angle* -2δ *needed to enforce the flow to be parallel to surface* 3*. Finally, a recompression is needed to obtain an horizontal flow parallel to surface* 4*, after an upward flow deflection of angle* δ*.*

As a consequence of the explained scenario, the pressure coefficients are thus defined:

$$C_{p,1} = 0 \tag{2}$$

$$C_{p,2} = C_{p,1} + \frac{2\delta}{\sqrt{M_\infty^2 - 1}} = 0.0931 \tag{3}$$

$$C_{p,3} = C_{p,2} + \frac{2(-2\delta)}{\sqrt{M_\infty^2 - 1}} = -0.0931 \tag{4}$$

$$C_{p,4} = C_{p,3} + \frac{2\delta}{\sqrt{M_\infty^2 - 1}} = 0. \tag{5}$$

Problem 2. *The same bump of Problem 1, of length* L*, is placed in a duct of height* $L/2$*, with an upstream Mach number* $M_\infty = 1.25$ *(Figure 1b). Evaluate, according to the linear small perturbation theory, the distribution of the pressure coefficient on the top and bottom surfaces for* $x/L \in (0, 1)$*, where the origin is placed at the feet of the ramp.*

Solution 2.

- *Bottom wall:* $C_p = 0.0931$ *for* $x/L \in (0, 0.5)$, $C_p = -0.0931$ *for* $x/L \in (0.5, 0.75)$, $C_p = 0.0931$ *for* $x/L \in (0.75, 1)$
- *Top wall:* $C_p = 0$ *for* $x/L \in (0, 0.375)$, $C_p = 0.1862$ *for* $x/L \in (0.375, 0.875)$, $C_p = -0.1862$ *for* $x/L \in (0.875, 1)$

Problem 3. *Consider a diamond-shaped profile with an opening angle* δ *of* 4°*. Determine for an upstream Mach number* $M_\infty = 2$ *at a null incidence:*

- *the pressure coefficient on each surface by means of the linear small perturbation theory;*
- *the lift and drag on the basis of the pressure coefficients found;*
- *the lift and drag coefficients with the small perturbation theory direct formulas.*

Solution 3.

- $C_{p,2} = 0.0403$, $C_{p,3} = -0.0403$, $C_{p,4} = 0.0403$, $C_{p,5} = -0.0403$;
- $C_l = 0$, $C_d = 0.002814$;
- $C_l = 0$, $C_d = 0.002816$;

Assignment 4: Small Perturbations

Compressible Flows - Master Course in Space and Astronautical Engineering

Sapienza University of Rome

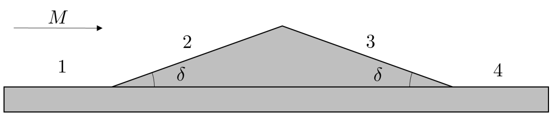
Problem 4. *Consider a diamond-shaped profile with an opening angle* $\delta = 4°$*, a chord* $c = 1\,m$ *at an angle of attack* $\alpha = 0°$ *and at a distance from a bottom wall equal to* $h = 3.4614\,c$*. (Figure 1c) For an upstream Mach number* $M_1 = 2.00$*, evaluate the position of the impingement of the compression and expansion waves originating from the airfoil at the wall.*

Solution 4. *1st compression wave:* $x_1 = 5.995\,m$*, expansion wave:* $x_2 = 6.465\,m$*, 2nd compression wave:*$x_3 = 6.995\,m$*.*

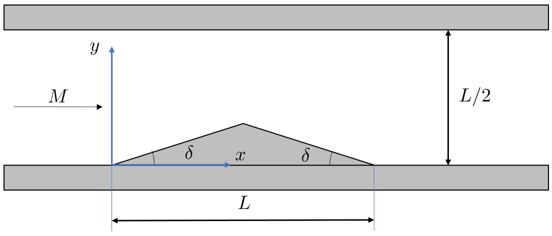
Problem 5. *Two identical diamond-shaped profiles with an opening angle* $\delta = 4°$ *and chord* c *are at a distance* $h = 3.4614\,c$ *from a bottom wall and are separated by a distance* $d = 10.46\,c$ *(Figure 1d). For an upstream Mach number* $M_\infty = 2$ *at a null incidence, determine the lift and drag coefficients on the basis of the pressure coefficients found by means of the linear small perturbation theory.*

Solution 5.

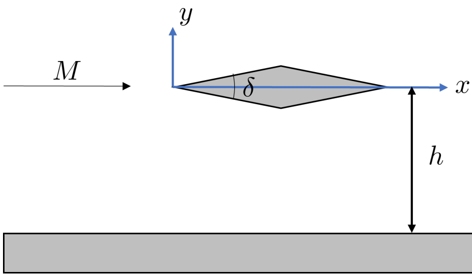
$C_{l,1} = 0,\ C_{d,1} = 0.002814,\ C_{l,2} = 0.01008,\ C_{d,2} = 0.001056$



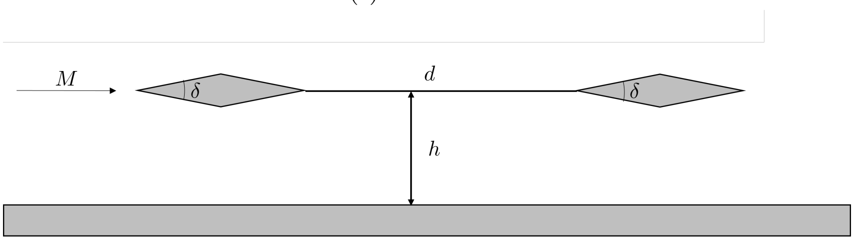
(a) Problem 1.



(b) Problem 2.



(c) Problem 4.



(d) Problem 5.

Figure 1: Problems figures.

References

[1] Renato Paciorri. *Esercizi di Gasdinamica*. Edizioni Ingegneria 2000, 2000.