Assignment 5: Oblique Shocks and Expansion Waves

Compressible Flows - Master Course in Space and Astronautical Engineering

Sapienza University of Rome

Problem 1. *Supersonic air at* $M_1 = 2.0$ *and* $p_1 = 75.0\,kPa$ *impinges on a two-dimensional edge of half-angle* $\delta = 10°$ *(Figure 1d). Calculate the two possible oblique shock angles,* β_{weak} *and* β_{strong}*, that could be formed by this wedge. For each case, calculate the pressure and Mach number downstream of the oblique shock, compare, and discuss.*

Solution 1. *The flow is steady and the boundary layer is thin. As a result, we can approximate the oblique shock deflection as equal to the wedge half-angle* $\delta = 10°$.
From the oblique shock tables, we obtain that in correspondence of the Mach number M_1 *and of the semi-opening angle of the wedge* δ*, the two solutions are:*

$$\sigma_{weak} = 39.31° \quad - \quad \sigma_{strong} = 83.70°\,. \tag{1}$$

By using these values, we find the upstream, normal Mach numbers:

$$M_{1n,weak} = M_1\, sin(\sigma_{weak}) \approx 1.27 \quad - \quad M_{1n,strong} = M_1\, sin(\sigma_{strong}) \approx 1.99 \tag{2}$$

By searching these two Mach number in the normal shock tables, we find that:

$$M_{2n,weak} = 0.80165, \quad (p_2/p_1)_{weak} = 1.7150 \quad - \quad M_{2n,strong} = 0.57907, \quad (p_2/p_1)_{strong} = 4.4534 \tag{3}$$

Thus, we can obtain the static pressure after the weak and strong oblique shocks as:

$$p_{2,weak} = (p_2/p_1)_{weak} p_1 = 129\,kPa \quad - \quad p_{2,strong} = (p_2/p_1)_{strong} p_1 = 334\,kPa. \tag{4}$$

Finally, we calculate the downstream Mach numbers as equal to:

$$M_{2,weak} = \frac{M_{2n,weak}}{sin(\sigma_{weak} - \delta} \approx 1.64 \quad - \quad M_{2,strong} = \frac{M_{2n,strong}}{sin(\sigma_{strong} - \delta} \approx 0.60\,. \tag{5}$$

We thus notice that the changes imposed by a strong oblique shock both in pressure and Mach number are much grater than the ones across a weak oblique shock.

Problem 2. *Supersonic air at* $M_1 = 2.0$ *and* $230\,kPa$ *flows parallel to a flat wall that suddenly expands by* $\delta = 10°$ *(Figure 1e). Ignoring any effects caused by the boundary layer along the wall, calculate the downstream Mach number* M_2 *and pressure* p_2.

Solution 2. $M_2 = 2.38$, $p_2 = 126\,kPa$.

Problem 3. *Air flowing at* $32\,kPa$, $240\,K$, *and* $M_1 = 3.6$ *is forced to undergo an expansion turn of* $15°$. *Determine the Mach number, pressure, and temperature of the air after the expansion.*

Solution 3. $M_2 = 4.81$, $p_2 = 6.65\,kPa$, $T_2 = 153\,K$

Problem 4. *Consider the supersonic flow of air at upstream conditions of* $70\,kPa$ *and* $260\,K$ *and a Mach number of* 2.4 *over a two-dimensional wedge of half-angle* $10°$. *If the axis of the wedge is tilted* $25°$ *with respect to the upstream airflow (Figure 1c), determine the downstream Mach number, pressure, and temperature* above *the edge.*

Solution 4. $M_2 = 3.105$, $p_2 = 23.8\,kPa$, $T_2 = 191\,K$.

Problem 5. *Considering the same geometrical conditions of Problem 4. What is the Mach number, pressure and temperature* below *the edge if the upstream Mach number is* $M_1 = 5$?

Solution 5.
Weak sol.: $M_2 = 1.72$, $p_2 = 1.168\,MPa$, $T_2 = 973\,K$
Strong sol.: $M_2 = 0.61$, $p_2 = 1.949\,MPa$, $T_2 = 1458\,K$

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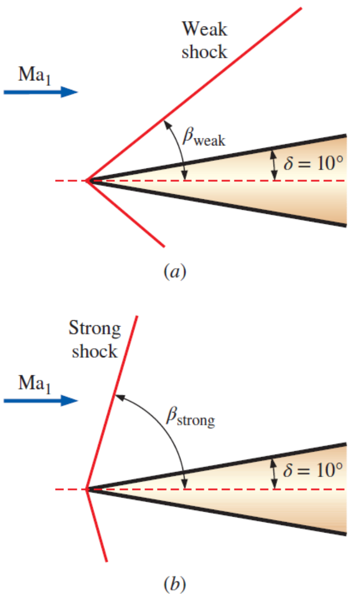
Problem 6. *Consider the double ramp in Figure* **??** *with* $\delta_1 = 2^\circ$, $\delta_2 = 5^\circ$, $l = 1\,m$, *and an incoming flow with undisturbed Mach number* $M_\infty = 5$. *Determine the coordinates of the intersection between the two oblique shocks that are generated in a frame whose origin is centred at the foot of the first ramp.*

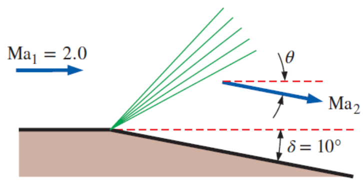
Solution 6. $x = 4.1142\,m$, $y = 0.9385\,m$.

Problem 7. *Determine the maximum value of the deviation* δ *for which the flow in Figure* **??** *remains attached for an inflow Mach number* $M = 4$.

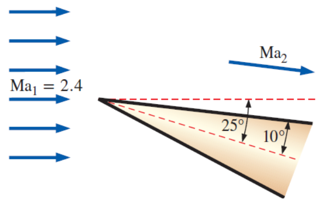
Solution 7. $\delta = 64.67^\circ$.



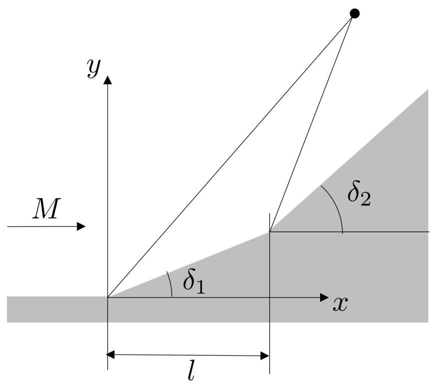
(a) Problem 1.



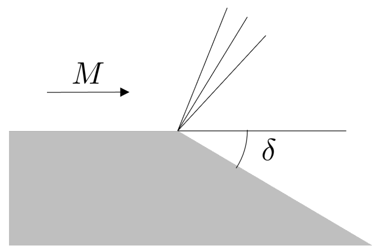
(b) Problem 2.



(c) Problem 4.



(d) Problem 6.



(e) Problem 7.

Figure 1: Problems figures.

References

[1] YA Cengel and JM Cimbala. *Fluid Mechanics. Fundamentals and Applications.* New York: McGraw-Hill, 2018.