Assignment 6: Compressible Flow over Airfoils

Compressible Flows - Master Course in Space and Astronautical Engineering

Sapienza University of Rome

Problem 1. *Consider a supersonic diamond profile with a semi-opening angle $\delta = 4°$ at an angle of attack $\alpha = 2°$. For an upstream Mach number $M_1 = 4.00$, evaluate:*

- *the lift and drag coefficients with the small perturbation theory;*
- *the pressure coefficient and Mach number on each surface, according to the numbering in Figure 1a;*
- *the lift and drag coefficients with the exact theory.*

Solution 1. *According to the small perturbation theory, the lift and drag coefficients are:*

$$C_l = \frac{4\alpha}{\sqrt{M_1^2 - 1}} = 0.0361 \quad C_d = \frac{4}{\sqrt{M_1^2 - 1}} \left(\alpha^2 + tan^2\delta\right) = 0.00631\,. \tag{1}$$

We now discuss the flow features between the different regions, and we determine the corresponding pressure coefficients and Mach numbers.

Region 1 - 2 *On the top, the increase in slope of the profile with respect to the direction of the flow induces a weak oblique shock wave with an angle $\beta = \delta - \alpha = 2°$. From the weak oblique shock waves tables, for $M_1 = 4.00$ and $\beta = 2°$, we find that:*

$$\sigma = 15.81° \quad \Rightarrow \quad M_{1n} = M_1 sin(\sigma) \approx 1.09 \tag{2}$$

In correspondence of the normal Mach number just found, the normal shock wave tables say that we have:

$$M_{2n} = 0.91965 \quad p_2/p_1 = 1.2194\,. \tag{3}$$

Hence, we have that the Mach number and pressure coefficient in region 2 are equal to:

$$M_2 = \frac{M_{2n}}{sin(\sigma - \beta)} \approx 3.85 \quad C_{p,2} = \frac{\frac{p_2}{p_1} - 1}{\frac{1}{2}\gamma M_1^2} = 0.0196\,. \tag{4}$$

Region 2 - 3 *In order to follow the surface of the profile from region 2 to region 3, the flow must undergo an isentropic expansion. Given the conservation of the first Riemann invariant across this expansion, we obtain:*

$$R_1 = \theta_2 + \omega(M_2) = \theta_3 + \omega(M_3) \quad \Rightarrow \quad \omega(M_3) = \omega(M_2) + \theta_2 - \theta_3\,. \tag{5}$$

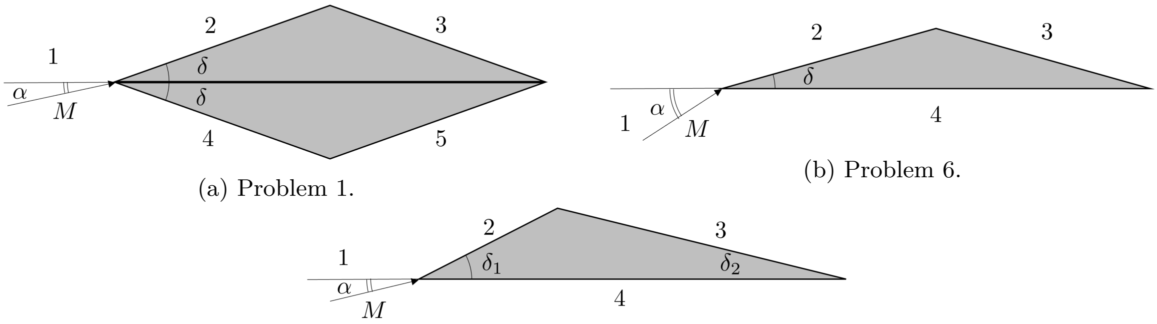
Since $\theta_2 - \theta_3 = 2\delta = 8°$, and $\omega(M_2 \approx 3.86) = 63.89°$, it follows that:

$$\omega(M_3) = 71.89° \quad \Rightarrow \quad M_3 \approx 4.50\,. \tag{6}$$

From the isentropic tables, in correspondence of M_3 we obtain the value of $p_3/p_0 = 0.0034553$, and thus we obtain that:

$$\frac{p_3}{p_1} = \frac{p_3}{p_0}\frac{p_0}{p_2}\frac{p_2}{p_1} = 0.52275 \quad \Rightarrow \quad C_{p,3} = \frac{\frac{p_3}{p_1} - 1}{\frac{1}{2}\gamma M_1^2} = -0.042612\,. \tag{7}$$

Be careful to the total pressure in region 2 and 3 in the above formula. The presence of a shock changes the value of the total pressure, and thus the p_0 indicated is more precisely $p_{02} < p_{01}$.



(a) Problem 1.

(b) Problem 6.

(c) Problem 7.

Figure 1: Problems figures.

Region 1 - 4 Similarly to region 3, the decrease in slope in the bottom part of the profile induces a weak oblique shock wave with $M_1 = 4$ but with $\beta = \delta + \alpha = 6°$. From the weak oblique shock waves tables, for $M_1 = 4.00$ and $\beta = 6°$, we find that:

$$\sigma = 18.81° \quad \Rightarrow \quad M_{1n} = M_1 sin(\sigma) \approx 1.29 \tag{8}$$

In correspondence of the normal Mach number just found, the normal shock wave tables say that we have:

$$M_{4n} = 0.79108 \quad p_4/p_1 = 1.7748\,. \tag{9}$$

Hence, we have that the Mach number and pressure coefficient in region 2 are equal to:

$$M_4 = \frac{M_{4n}}{sin(\sigma - \beta)} \approx 3.57 \quad C_{p,2} = \frac{\frac{p_4}{p_1} - 1}{\frac{1}{2}\gamma M_1^2} = 0.0692\,. \tag{10}$$

Region 4 - 5 In order to follow the surface of the profile from region 4 to region 5, the flow must undergo an isentropic expansion. Given the conservation of the *second* Riemann invariant across this expansion, we obtain:

$$R_2 = \theta_4 + \omega(M_4) = \theta_5 - \omega(M_5) \quad \Rightarrow \quad \omega(M_5) = \omega(M_4) + \theta_5 - \theta_4\,. \tag{11}$$

Since $\theta_5 - \theta_4 = 2\delta = 8°$, and $\omega(M_4 \approx 3.56) = 59.47°$, it follows that:

$$\omega(M_5) = 67.47° \quad \Rightarrow \quad M_5 \approx 4.14\,. \tag{12}$$

From the isentropic tables, in correspondence of M_5 ($\approx$ 4.15) we obtain the value of p_5/p_0 = 0.0054028, and thus we obtain that:

$$\frac{p_5}{p_1} = \frac{p_5}{p_0}\frac{p_0}{p_4}\frac{p_4}{p_1} = 0.785 \quad \Rightarrow \quad C_{p,5} = \frac{\frac{p_5}{p_1} - 1}{\frac{1}{2}\gamma M_1^2} = -0.0192\,. \tag{13}$$

Be careful to the total pressure in region 4 and 5 in the above formula. The presence of a shock changes the value of the total pressure, and thus the p_0 indicated is more precisely $p_{04} < p_{01}$.

Finally, the lift and drag coefficients for a supersonic diamond profile are:

$$C_l = \frac{1}{2cos(\delta)}\left[(C_{p,5} - C_{p,2})cos(\alpha - \delta) + (C_{p,4} - C_{p,3})cos(\alpha + \delta)\right] \approx 0.0363\,, \tag{14}$$

$$C_d = \frac{1}{2cos(\delta)}\left[(C_{p,5} - C_{p,2})sin(\alpha - \delta) + (C_{p,4} - C_{p,3})sin(\alpha + \delta)\right] \approx 0.00654\,. \tag{15}$$

As we can see, the values produced by the small perturbation theory gave us a rather good approximation in the case of limited Mach numbers and small angles.

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Problem 2. *A thin airfoil develops null lift at* $-1.1°$ *in an incompressible flow. Determine the lift coefficient of the same airfoil for an incidence equal to* $4°$ *and a Mach number equal to* 0.6.

Solution 2. $C_l = 0.6$.

Problem 3. *A thin symmetric airfoil flights at Mach number equal to* 0.7. *Determine the lift coefficient as a function of the incidence and the percentage increase of the lift slope* $\mathrm{d}C_l/\mathrm{d}\alpha$ *with respect to the incompressible case.*

Solution 3. $C_l = 8.8\,\alpha$, $\mathrm{d}C_l/\mathrm{d}\alpha(M_\infty = 0.7)$ *is* $40\,\%$ *higher than* $\mathrm{d}C_l/\mathrm{d}\alpha(M_\infty = 0)$.

Problem 4. *Consider a supersonic diamond profile with a semi-opening angle* $\delta = 3°$ *at an angle of attack* $\alpha = 3°$. *For an upstream Mach number* $M_1 = 4.00$, *evaluate:*

- *the lift and drag coefficients with the small perturbation theory;*
- *the pressure coefficient and Mach number on each surface, according to the numbering of Prob.1;*
- *the lift and drag coefficients with the exact theory.*

Solution 4.

- $C_l = 0.0541$, $C_d = 0.00568$;
- $C_{p,2} = 0.00$; $C_{p,3} = -0.0424$; $C_{p,4} = 0.0692$; $C_{p,5} = -0.00384$;
  $M_2 = 4.00$; $M_3 = 4.50$; $M_4 = 3.57$; $M_5 = 3.98$;
- $C_l = 0.0536$, $C_d = 0.00584$;

Problem 5. *Consider a supersonic diamond profile with a semi-opening angle* $\delta = 4°$ *at an angle of attack* $\alpha = 6°$. *For an upstream Mach number* $M_1 = 4.00$, *evaluate:*

- *the lift and drag coefficients with the small perturbation theory;*
- *the pressure coefficient and Mach number on each surface, according to the numbering of Prob.1;*
- *the lift and drag coefficients with the exact theory.*

Solution 5.

- $C_l = 0.108$, $C_d = 0.0164$;
- $C_{p,2} = -0.0160$; $C_{p,3} = -0.0605$; $C_{p,4} = -0.133$; $C_{p,5} = 0.0206$;
  $M_2 = 4.16$; $M_3 = 4.88$; $M_4 = 3.29$; $M_5 = 3.82$;
- $C_l = 0.114$, $C_d = 0.0175$;

Problem 6. *Consider a supersonic triangular profile with an opening angle* $\delta = 3°$ *at an angle of attack* $\alpha = 7°$. *For an upstream Mach number* $M_1 = 5.00$, *evaluate:*

- *the pressure coefficient and Mach number on each surface, according to the numbering of Figure 1b;*
- *the lift and drag coefficients with the exact theory.*

Solution 6.

- $C_{p,2} = -0.0571$; $C_{p,3} = -0.0571$; $C_{p,4} = 0.0711$;
  $M_2 = 5.45$; $M_3 = 6.30$; $M_4 = 4.30$;;
- $C_l = 0.127$, $C_d = 0.0156$;

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Problem 7. *Consider the supersonic triangular profile in Figure 1c with* $\delta_1 = 5°$ *and* $\delta_2 = 3°$ *at an angle of attack* $\alpha = 2°$*. For an upstream Mach number* $M_1 = 8.00$*, evaluate:*

- *the pressure coefficient on each surface, according to the numbering on the figure;*
- *the lift and drag coefficients with the exact theory.*

Solution 7.

- $C_{p,2} = 0.01696$; $C_{p,3} = -0.01459$; $C_{p,4} = 0.01037$;
- $C_l = 0.0131$, $C_d = 0.001492$;

References

[1] John Anderson. *Fundamentals of aerodynamics*. Tata McGraw-Hill Education, 2017.

[2] Giorgio Graziani. *Aerodinamica*. Rome: Casa Editrice Università La Sapienza, 2007.

[3] Renato Paciorri. *Esercizi di Gasdinamica*. Edizioni Ingegneria 2000, 2000.