Assignment 7: Newtonian Theory

Compressible Flows - Master Course in Space and Astronautical Engineering

Sapienza University of Rome

Problem 1. *Consider a blunt body with a frontal section and two lateral sides with an angle δ of $\pi/4\,rad$ (as in Figure 1a), at a null angle of attack and at $M = 5$. Evaluate, using both Newtonian and modified Newtonian theory:*

- *the pressure coefficient of each section;*
- *the drag coefficient based on the length of the rear section l.*

Solution 1. *According to the Newtonian theory, the pressure coefficient on the frontal sections is:*

$$C_{p_i} = C_p^{max} sin^2(\theta_i) \tag{1}$$

where $i = 1, 2, 3$ identifies the surface with a slope relative to the flow *equal to θ_i, and where C_p^{max} is equal to* 2 *in the case of the classic Newtonian theory and is a function of the Mach number in the case of the modified Newtonian theory. Since the angle of attack is null, θ_i coincides with the geometrical slope of each surface with respect to the horizontal direction, and so: $\theta_1 = \pi/2$, $\theta_2 = -\theta_3 = \pi/4$. As a result, the pressure coefficients of the frontal sections are:*

$$C_{p_1} = C_p^{max} \quad C_{p_2} = C_{p_3} = \frac{C_p^{max}}{2}\,. \tag{2}$$

For the case of the Newtonian theory, $C_{p_1} = 2\,, C_{p_2} = C_{p_3} = 1$ For what concerns the back surface, the pressure coefficient is zero *($C_{p_4} = 0$), since the flow separates in correspondence of the top and bottom corner and thus the pressure is approximately equal to the reference pressure.*
In order to estimate the drag coefficient, we have to consider the projection of the pressure force (per unit span, since the problem is bidimensional) in the drag direction on each side of the body. Thus, if we indicate with n_i the normal to each surface, we have:

$$C_d = \frac{D}{1/2\rho U_\infty^2 l} = \tag{3}$$

$$= \frac{-p_1 n_1 \cdot \hat{D}\, l_1 - p_2 n_2 \cdot \hat{D}\, l_2 - p_3 n_3 \cdot \hat{D}\, l_3 - p_4 n_4 \cdot \hat{D}\, l_4}{1/2\rho U_\infty^2 l} \tag{4}$$

Since $\hat{D} = \hat{x}$ and the pressure coefficient $C_p = (p - p_\infty)/(1/2\rho U_\infty^2)$ of the back surface is zero for such blunt bodies, it is possible to demonstrate that the equation above is equivalent to adding the pressure coefficient of each front surface weighted by the adimensional projection of the length of the corresponding edge in the streamwise direction. Thus, given also the simmetry of the problem, and given that $l_1 = l/2$ and $l_1 = \sqrt{2}\,l/4$, in our case we have that:

$$C_D = C_{p_1}\frac{l_1}{l} sin\left(\frac{\pi}{2}\right) + 2C_{p_2}\frac{l_2}{l} sin\left(\frac{\pi}{4}\right) = \frac{1}{2}(C_{p_1} + C_{p_2})\,. \tag{5}$$

Given Equation 1, the Newtonian theory gives a drag coefficient based on the rear $C_D = 1.5$.
If the modified Newtonian theory is adopted, the maximum pressure coefficient in Equation 1 must be evaluated according to the formula:

$$C_p^{max} = \frac{\frac{p_{02}}{p_\infty} - 1}{1/2\gamma M_\infty^2} = \frac{\frac{p_{02}}{p_{01}}\frac{p_{01}}{p_\infty} - 1}{1/2\gamma M_\infty^2}\,, \tag{6}$$

where p_{01} is the total pressure of the undisturbed flow, and p_{02} is the total pressure after a normal shock wave caused by a flow with Mach number $M = 5$. From the isentropic tables, at $M = 5$ we have $p_\infty/p_{01} = 0.00189$, while from the normal shock wave tables, at $M = 5$ we have $p_{02}/p_{01} = 0.0617$. As a result, we have $C_p^{max} = 1.808$. From Equation 2 and Equation 5, we obtain that:

$$C_{p_1} = 1.808 \quad C_{p_2} = C_{p_3} = 0.904 \quad C_D = 1.3560\,. \tag{7}$$

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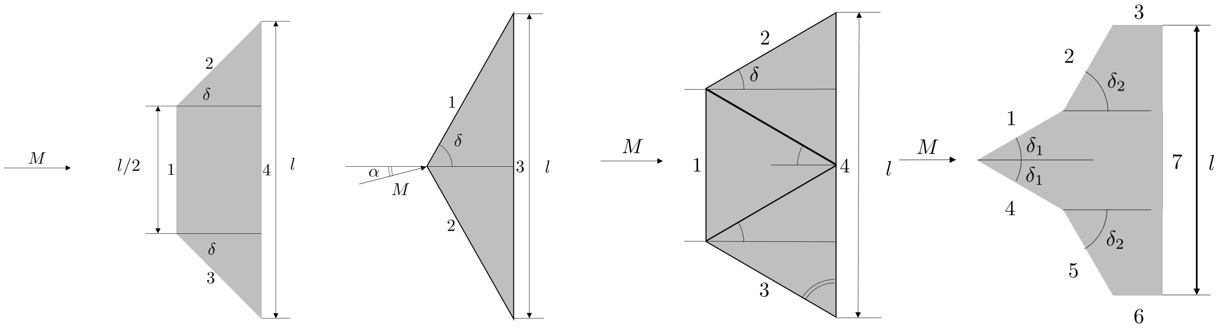

(a) Problem 1. (b) Problem 2. (c) Problem 3. (d) Problem 4.

Figure 1: Problems figures.

Problem 2. *Consider the blunt body in Figure 1b, where $\delta = 60°C$, $\alpha = 30°C$ and $M = 7$. Evaluate, using both Newtonian and modified Newtonian theory, the pressure coefficient of each section.*

Solution 2. $C_{p_1} = 0.5 - 0.4559$, $C_{p_2} = 2 - 1.8236$, $C_{p_3} = 0$

Problem 3. *Consider a blunt body with a frontal section and two lateral sides with an angle δ of $\pi/6\,rad$ (as in Figure 1c), at a null angle of attack and at $M = 7$. Evaluate, using both Newtonian and modified Newtonian theory:*

- *the pressure coefficient of each section;*
- *the drag coefficient based on the length of the rear section l.*

Solution 3.

- $C_{p_1} = 2 - 1.824$, $C_{p_2} = C_{p_3} = 0.5 - 0.456$, $C_{p_4} = 0$
- $C_D = 1.25 - 1.14$

Problem 4. *Consider the symmetric blunt body in Figure 1d with $\delta_1 = 30°$ and $\delta_2 = 60°$, at a null angle of attack and at $M = 7$. Evaluate, using both Newtonian and modified Newtonian theory ($\gamma = 1.4$):*

- *the pressure coefficient of each section;*
- *the drag coefficient based on the length of the rear section l.*

Solution 4.

- $C_{p,1} = C_{p,4} = 0.5 - 0.456$, $C_{p,2} = C_{p,5} = 1.5 - 1.368$, $C_{p,3} = C_{p,6} = C_{p,7} = 0$
- $C_D = 1.167 - 1.064$