Compressible Flows - 12/10/2016

(1) Consider a tank with an internal pressure p=1.2 atm and an external pressure of 1atm. The thank has attached a pure converging nozzle as in the picture (1). Verify under these conditions if the flow in the exit section is subsonic, sonic or supersonic.

From isentropic flow tables: For M=1, $p/p_0 = 0.528$, at M=1 $p=p^*$, therefore $p^*=0.528\ p_0 = 0.634$atm
$P_{exit}>p^*$ the flow is subsonic

(2) Calculate Mach number in the exit section.

$p/p_0=1/1.2=0.833 \Rightarrow M_{exit}=0.51$

(3) Considering that the initial temperature of the fluid in the tank is 20°C, calculate temperature and density of the fluid at the exit section

$T_0=(20+273)$°K; being at M=0.51 $T/T_0=0.95 \Rightarrow T=278$°K
$\rho = p/RT$, being p=1 atm = 1 10^5Pa, with R=286.9 J/kg°K for air => $\rho = 1.25$ kg/m^3

(4) Considering that exit section has an area $A_{exit}= 1m^2$, calculate Mach number at an intermediate section B with an area $A_B=1.2m^2$

At the exit sect: M=0.51, $A_{exit}/A^*=1.32$, $A_B/A^* = (A_B/A_{exit})(A_{exit}/A^*) = (1.2)(1.32) = 1.584 \Rightarrow M=0.40$

(5) Suppose now, to reduce the external pressure p_{ext} to 0.634atm, what happens?

Being $p_{exit}=p^*$ in the exit section the flow is sonic (M=1)

(6) Suppose to reduce p_{ext} more, what happens?

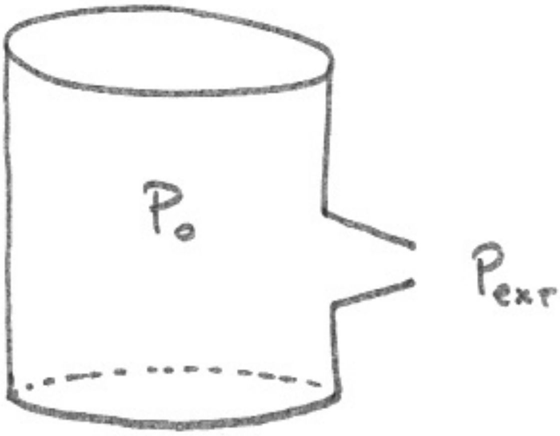
Nothing …. The nozzle is simply a converging nozzle and can't be supersonic, therefore the flow remains subsonic along all the converging nozzle and become sonic at the exit section.
Obviously outside, after the exit section, the residual pressure jump generates an additional non isentropic expansion.

(7) Suppose now to add a divergent part to the nozzle (fig 2). Being the throat area $A_{throat}=1m^2$ and external pressure $p_{ext}=0.5$atm, calculate the appropriate area of exit section and so that the converging-diverging nozzle is working under design conditions. Calculate Mach number at exit section.

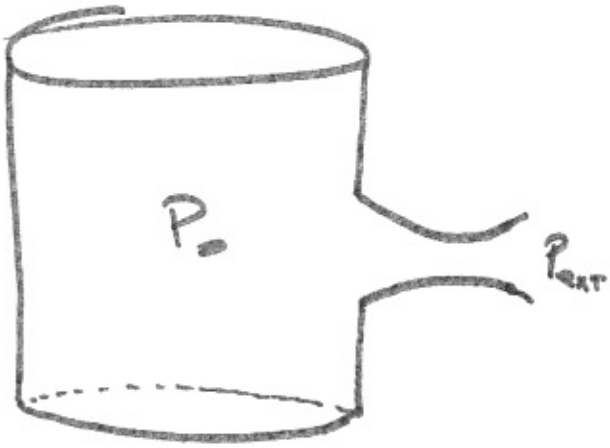
$p_{ext}/p_0 = 0.5/1.2 = 0.4167 \Rightarrow M=1.19 \Rightarrow A/A^*=1.0276 \Rightarrow A_{exit}= 1.0276m^2$

(8) Calculate temperature and density at exit section (initial gas temperature in the tank is 20°C)

$M=1.19 \Rightarrow T/T_0 = 0.779 \Rightarrow T=228$°K
$\rho = p/RT$, being p=0.5 atm = 0.5 10^5Pa => $\rho = 0.76$ kg/m^3



(1)



(2)