Compressible Flows
Exam – 27/03/2021

1) [8 pts] Consider a converging-diverging nozzle fed by a tank containing a gas with constant $R = 297\,J/kgK$ and heat capacity ratio $\gamma = 1.4$. Stagnation pressure and temperature inside the tank are $p_0 = 1.0 \cdot 10^6\,Pa$, $T_0 = 1500\,K$ and the exit area is $A_e = 1.20\,m^2$. Considering that, under design conditions, the Mach number at the exit section is $M_e = 3.0$, find:

a) the value of the throat area: 0.28 m^2
b) the static pressure at the exit section: $2.72 \cdot 10^4$ Pa
c) the static temperature at the exit section: 535.71 K
d) the density at the exit section: 0.17 kg/m^3
e) the overall mass flow rate through the nozzle: 290.71 kg/s

the three external pressure values that characterize the nozzle regimes, i.e.:

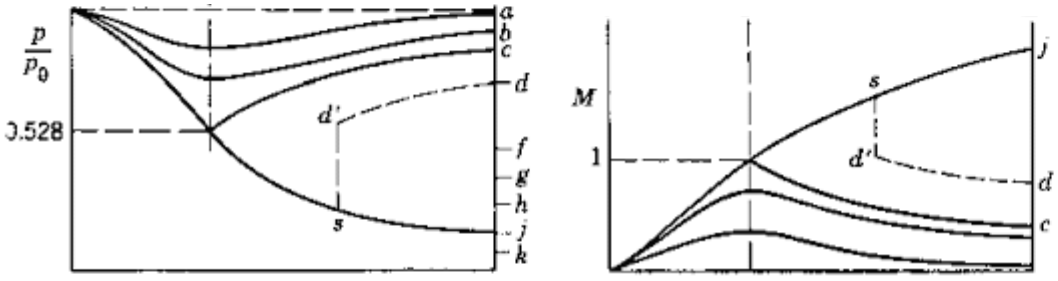
f) limit subsonic: $9.87 \cdot 10^5$ Pa
g) design conditions (limit supersonic): $2.72 \cdot 10^4$ Pa
h) over-expanded: $2.81 \cdot 10^5$ Pa

2) [6 pts] Consider a shock in the converging-diverging nozzle of ex. #1 at a section with area equal to $A_s = 0.902\,m^2$ in the diverging part of the nozzle. Find:

a) the Mach number before and after the shock: $M_1 = 2.70$ $M_2 = 0.50$
b) the static pressure before and after the shock: $p_1 = 4.30 \cdot 10^4$ Pa $p_2 = 3.58 \cdot 10^5$ Pa
c) the exit Mach number: $M_{e,s} = 0.35$
d) the exit static pressure: $p_{e,s} = 3.89 \cdot 10^5$ Pa

Finally, draw a sketch of the static pressure and of the Mach number along the converging-diverging nozzle if a shock is present inside the diverging section of the nozzle.

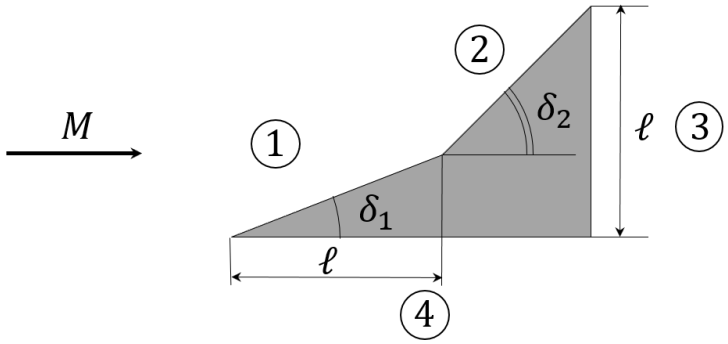
Condition **d** in the figures below.



3) [4 pts] Consider the blunt body shown in the figure below, where $\delta_1 = 30°$ and $\delta_2 = 45°$. With an upstream Mach number $M_\infty = 6.0$.
Evaluate using the Newtonian **or** the modified Newtonian theory:

a) the pressure coefficients on each section: 0.5/0.45 – 1.0/0.91 – 0.0 – 0.0
b) the drag coefficient based on the length of the rear section: 0.71/0.65.

Finally, evaluate the maximum pressure coefficient according to the modified Newtonian theory: 1.82.



4) [12 pts] Consider a supersonic profile with a semi-opening angle δ of 5.0°, at an angle of attack α of 5° and a Mach number $M_\infty = 5.50$. Evaluate:

a) C_L and C_D with the exact theory: 0.073 – 0.013
b) C_L and C_D with small perturbation theory: 0.065 – 0.011
c) Mach number on profile surface #2: 5.50
d) Pressure coefficient (C_p) on profile surface #2: 0.00
e) Mach number on profile surface #3: 7.05
f) Pressure coefficient (C_p) on profile surface #3: -0.037
g) Mach number on profile surface #4: 4.32
h) Pressure coefficient (C_p) on profile surface #4: 0.11
i) Mach number on profile surface #5: 5.30
j) Pressure coefficient (C_p) on profile surface #5: $6.80 \cdot 10^{-4}$