The Analysis Centric Isentropic Compression Waves

1Mikhail Pavlovich Bulat and 2Pavel Victorovich Bulat

1Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics, Saint-Petersburg, Russia
2Business Computer Center", Saint-Petersburg, Russia

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Abstract: In this article we consider the problem of isentropic compression of supersonic flow. If you specify the shape of the concave surface in accordance with the equation of the streamlines in a Prandtl-Meyer plane wave, then the inleakage of supersonic flow on it forms a centered compression wave, the characteristics of which intersect at one point. Shock-wave structure of the main shock wave and mirrored gas-dynamic discontinuity forms at this point. This article examines the formation of this structure.

Key words: Diffuser isentropic compression • Centered wave of compression • Shock-wave structure • The polar compression

INTRODUCTION

If we define the shape of the concave surface in accordance with the equation equation of streamlines in the Prandtl-Meyer plane wave [1], then compression waves (characteristics) of the shock-wave structure \( \omega_p \) are concurrent (A, Fig. 1) in the event of leakage of supersonic flow on it. This forms a shock wave structure with the main shock of finite intensity \( \sigma \) and reflected gas-dynamic discontinuity \( R \), which can be a shock wave, centered rarefaction wave or a 2nd-order weak discontinuity (discontinuous characteristic, in which derivative values of the gas dynamics variables have a discontinuity) [2, 3].

The Analysis of Centered Isentropic Compression Waves: It is known that in the compression wave flow parameters are described by Prandtl-Meyer solution [4] for a plane centered wave

\[
\omega_\infty + \vartheta_t = \omega_\infty + \vartheta_\omega
\]

where \( \omega \) - Prandtl-Meyer function
\( \theta \) – velocity vector inclination angle.

Then, introducing the notion of the compression wave intensity \( J = P_0 / P_\infty \), we can write

\[
\text{Shocked wave}
\]

\[
\text{Ig} = \frac{(J_m - 1)}{(J_m + \varepsilon)} \quad \frac{(1 - \varepsilon)(J - 1)}{(J_m + \varepsilon) - (1 - \varepsilon)(J - 1)}
\]

where

Corresponding Author: Mikhail Pavlovich Bulat, Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics, Kronverksky pr., 49, Saint-Petersburg, 197101, Russia.
$$J_m = (1 - \varepsilon)M^2 - \varepsilon; \; \varepsilon = \frac{\gamma - 1}{\gamma + 1}.$$  

Curves described by these equations will be termed compression polar and shock polar respectively [3]. The points on the compression polar show the relation between the pressure over the centered wave structure and the pressure in the undisturbed flow, as well as the rotation angle of the stream in the center of the compression wave. At the origin ([\(\lambda\)=0, [\(\beta\)=0]) these curves have at least second order of contact [5]. This property can be easily expressed as follows:

$$\beta_0^{(i)} = \beta_0^{(i)} + \Delta_i \beta_0^{(i)},$$

where

$$\Delta_1 = \Delta_2 = 0, \beta_0^{(i)} = \frac{d \beta}{d J_{,i}} = \frac{(M^2 - 1)^{1/2}}{\gamma M^2}.$$

The difference in the values of the higher derivatives (i>2) of these curves at J=1 depends on the \(\Delta\). Omitting elementary calculations, we can write for \(\Delta\) at I=3,4

$$\Delta_3 = \frac{((M^2 - 1)^{1/2} - 1)^2 - 4\varepsilon}{4(1 + \varepsilon)^2},$$

$$\Delta_3 = \frac{A}{M^2 - M^4} - \frac{B}{M^2 - 1} + \frac{C}{M^2 - 1} + \frac{D}{(M^2 - 1)^2} + \frac{E}{\gamma M^2} + \frac{F}{(M^2 - 1)^2} + G,$$

$$A = \frac{1}{2 \alpha} \left( \frac{14}{\gamma} + \frac{11}{\gamma^2} - 1 \right), \; B = \frac{12}{\gamma}, \; C = \frac{16}{\gamma^3},$$

where

$$D = \frac{1}{8} \left[ 17 - \frac{15}{\gamma} - \frac{25}{\gamma^2} - \frac{7}{\gamma^3} \right], \; E = \frac{1}{8} \left[ -4 - 13 + 11 - \frac{3}{\gamma} - \frac{3}{\gamma^2} \right],$$

$$F = \frac{1}{8} \left[ -13 - \frac{7}{\gamma} + \frac{25}{\gamma^2} + \frac{19}{\gamma^3} \right], \; G = \frac{1}{8} \left[ 23 + \frac{17}{\gamma} + \frac{7}{\gamma^2} \right].$$

When [\(\gamma\)=const relation \(\Delta_3(M)\) is non-monotonic. It has roots as the Mach number has a value of

$$M_{f,2} = \left\{ \frac{2}{5 - 3\gamma} \left[ 3 - \gamma \pm (\gamma^2 - 1)^{1/2} \right] \right\}^{1/2}.$$  

And at least at \(M = \sqrt{2}\) for any [\(\gamma\)].

If \(M = M_{f,2}\) polars (compression and impact) have a third order of contact at the origin. The function \(\Delta_3(M)\) has no real roots. The product \(\Delta_3 \beta_0^{(i)}\) has extremums at \(M = \sqrt{2}\) and \(M = M_{f,2}\).

$$M_{f,2} = \sqrt{2} \left\{ \frac{2(1 - \varepsilon) + \sqrt{3(1 - \varepsilon)}}{1 - 4\varepsilon} \right\}^{1/2}.$$  

\(\Delta_3 \beta_0^{(i)}\) tends to 8 as \(M \to 1\) and to 0 as \(M \to \infty\). The roots of the product coincide with the roots of \(\Delta_3\). From the above results it follows that compression polar at the origin can be held inside the shock polar \((M < M_{f,1}\) and \(M > M_{f,2}\)) and outside it \((M_{f,1} < M < M_{f,2}\)). The type of reflected discontinuity depends on the relative position of polars (Fig. 2).

In the design of the isentropic diffusor [3,6] it is necessary to set the concave surface geometry such a way that the reflected discontinuity R (Fig. 1) should became shock wave, then the shock wave configuration will have the property of structural stability [7]. The smaller the intensity of the shock, the higher the total pressure recovery coefficient in the diffuser \(\nu = P_s/P_{\text{inj}}\). There is a possible configuration in which shock R degenerates into a discontinuous characteristic (gas-dynamic discontinuity of the second order). Such CCW called neutral.

Neutral CCW: For some values of M and [\(\gamma\)] compression polar and shock polar can intersect. At the point of intersection we have the equality of the
intensities \((J_s = J_d)\) of the shock wave \(\sigma\) and CCW, as well as the rotation angles of the flow at these discontinuities. Consequently, the condition of collinearity of the velocity vectors on a tangential discontinuity is executed under degeneration of the reflected discontinuity \(R\) into a characteristic. We call such a SWS neutral, the intensity of the wave at the polar intersection point we denote \(J_s\) and the corresponding curve \(J_s(M, [\gamma])\) will be called the neutral polar (Fig. 3). Neutral polar has two branches. Since the isentropic compression wave cannot slow down the flow to a speed lower than the speed of sound, then the domain of its existence is bounded above by the sonic line \(J_{\sigma}\).

Figure 3 shows a graph of the shock wave \(J_s\) and the compression wave \(J_{\omega}\) sound intensity, as well as characteristic points \(M_i\) \((i=1,2,3,4)\), “s”, “w”.

The points \(M_{s1}\) and \(M_{s4}\) correspond to Mach numbers \(M_{f1}\) and \(M_{f2}\), analytically derived above.

\(J_s\) - centered compression wave intensity, \(J_{\omega}\) - centered compression wave sound intensity, \(J_\sigma\) – shock wave sound intensity, \(M_{s1}-M_{s4}\) – special Mach numbers, \(M_{s}\) - Mach number, which limits the domain of existence of shock-wave structures with mirrored discontinuity – shock wave.

\(M_{s}\) is the point of intersection of the left branch of the graph \(J_s(M)\) with the sonic line of the compression polar \(J_\sigma\) [\(\omega\)]. \(M_{s1}\) – fold point of the neutral polar \(J_s(M)\). Point «s» of intersection of \(J_s\) and \(J_\omega\) corresponds to the case where the neutral polar crosses the shock polar at the sonic point. The two branches of the neutral polar and the sonic line limit the domain of existence of SWS with the reflected discontinuity (shown shaded) from three sides. Special intensity is marked by index “w” in Figure 4 and its corresponding Mach number \(M_{\omega}\). At Mach number \(M_{\omega}\) shock polars issued by the points of the compression polar lying above the \(J_\omega\) cannot cross the main shock polar (Fig. 4).

Thus, for each of the adiabatic index, starting with the Mach number \(M_{\omega}\), SWS with the intensity of CCW greater than \(J_\omega\), containing the reflected shock wave cannot exist. At Mach numbers greater than \(M_{\omega}\) reflected shock wave is a wave of rarefaction for all values of the intensity of
Table 1:

<table>
<thead>
<tr>
<th>[gamma]</th>
<th>$M_{f1}$</th>
<th>$M_{f2}$</th>
<th>$M_{a1}$</th>
<th>$M_{a2}$</th>
<th>$M_{a3}$</th>
<th>$M_{a4}$</th>
<th>$J_a$</th>
<th>$\beta_{au}$</th>
</tr>
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<tbody>
<tr>
<td>1.1</td>
<td>1.302</td>
<td>1.666</td>
<td>1.302</td>
<td>1.486</td>
<td>-</td>
<td>1.666</td>
<td>2.599</td>
<td>6.345</td>
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<tr>
<td>1.25</td>
<td>1.265</td>
<td>2</td>
<td>1.265</td>
<td>1.484</td>
<td>1.940</td>
<td>2</td>
<td>2.990</td>
<td>8.713</td>
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<tr>
<td>1.4</td>
<td>1.245</td>
<td>2.539</td>
<td>1.245</td>
<td>1.478</td>
<td>2.230</td>
<td>2.539</td>
<td>3.483</td>
<td>12.18</td>
</tr>
<tr>
<td>1.67</td>
<td>1.225</td>
<td>-</td>
<td>1.225</td>
<td>1.746</td>
<td>2.857</td>
<td>-</td>
<td>4.670</td>
<td>22.76</td>
</tr>
</tbody>
</table>

CCW except $J_a$. Specific Mach numbers, CCW intensities corresponding to them and flow turning angles are given in Table 1.

CONCLUSION

CCW with the reflected shock wave discontinuity are suitable for the design of supersonic diffusers, as they have the property of structural stability, i.e. when external parameters are changed within certain limits, the SWS form does not change and the compression ratio of flow and total pressure recovery coefficient change gradually. This provides the required margin of self-regulation of diffuser. Domain of existence of such CCW is limited by Mach number $M_a = 3.483$ for air. Limiting case of the reflected shock is discontinuous characteristic. This case corresponds to a neutral CCW, which provides the highest degree of pressure increase and the recovery of the total pressure in the diffuser. In a fairly complete classification of SWS, resulting from the interaction of gasdynamic discontinuities proposed in [2,8,9,10], centered isentropic compression wave is absent. The above analysis is its complement.

Findings: The above analysis of properties of a centered compression wave complements the theory of stationary gas-dynamic discontinuities. Boundaries of the domains existence of the shock-wave structures that are optimal for the design of supersonic diffusers are defined also.

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