Optimal Shape Design of Supersonic, Mixed-Compression, Fixed-Geometry Air Intakes for SSTO Mission Profiles.

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The problem of maximizing the performance of a fixed-geometry air intake geometry of a vehicle accelerating over a wide range of flight Mach number is addressed. An extension of the Seddon-Goldsmith procedure is used to estimate the flow pattern involving a curved bowshock, a triple point interaction, and wall shock reflection which characterizes the subcritical regime of operations. The approximate model has been validated against detailed CFD calculations of the flowfield about the air intake. Finally, the approximate model is adopted to find the geometry which optimizes the fuel-to-mass ratio over a constant dynamic pressure trajectory.

Introduction

The air intake is the most critical part of a supersonic/hypersonic airbreathing propulsion system. It must deliver air to combustion chamber for a wide range of flight Mach number at a desired rate and flow conditions. This delivery must be accomplished by as little losses and drag as possible.

At high flight Mach number the compression process is accomplished by a succession of oblique shocks generated both ahead (external compression) and inside the converging part of the air intake (internal compression). The supersonic compression ends at a normal shock located at or downstream the intake throat and might be followed by further diffusion in the subsonic flow regime (ramjet).

The intake geometry needs to be properly shaped to achieve an optimal performance. This task is relatively manageable if the intake is to be operated at nearly constant freestream conditions, as it happens for a cruiser type vehicle. However, a reusable airbreather launcher, such as a SSTO vehicle, attains orbital speeds and heights by a continuous acceleration (accelerator type vehicle), which forces the air intake to operate over a wide range of flight Mach numbers and altitudes. To obtain an envelope of nearly optimal intake performance requires adopting a variable-geometry intake strategy, with a multi-point design selection along the ascent corridor.

The air intake during off-design conditions displays several flow regimes, classified as subcritical or supercritical according to whether the flow Mach number is respectively below or above the design Mach number. The subcritical regime is prompted by the need of reducing the massflow captured by the intake so as to match the reduced massflow requirement elaborated by the engine, reduction that is accomplished by spilling air over the intake cowl lip. Spilling might occur under both subsonic or supersonic flow conditions. Subsonic spillage involves the formation of a complex shock pattern ahead of the intake duct including a strong bow shock, whereas supersonic spillage can be accomplished by oblique shock waves alone.

The off-design performance might be largely lower than at the design point. Before resorting to a wholly variable-geometry intake strategy, it is important to understand how and under what conditions an optimal accelerator performance for a vehicle flying over a range of variable free stream conditions could be achieved by adopting a fixed-geometry intake.

Of special interest is to understand how to choose the design point for a fixed-geometry intake, its geometry and how its overall performance evaluated over a flight corridor can be predicted. Clearly, computational fluidodynamics (CFD) can be used to obtain the flowfields about the intake at different free-stream conditions for several intake geometries, and next derive the overall performance prediction from this database. This procedure is very time consuming and at the design stage it is preferable to adopt a "lighter" tool of analysis and to postpone the use of CFD tools in the verification stage of the design process.

In this paper, we illustrate a procedure to attain an intake shape optimal over a flight corridor whose core tool is an approximate, algebraic, model of the shock pattern developed during the subcritical operation regime.

This approximate shock model allows us to predict the intake off-design performance with a minimal computational effort thus allowing for a parametric investigation of very many intake shape designs and ultimately for an automatic design procedure to be implemented. Moreover, the simplification process followed to obtain the approximate shock model allows us to pinpoint the main mechanisms governing the interaction between the shock pattern ahead of the intake...
duct and the engine during off-design operations, thus enabling us to establish general intake design guidelines.

The paper is organized as follows. First a reference intake geometry will be introduced; then the optimal performance at on-design conditions of the reference intake will be parametrically analyzed and discussed. Next, the approximate, algebraic, model of the shock pattern developed during the subcritical operation regime will be illustrated. The following section will present the results of the validation of the simplified model, validation which will be carried out by comparing the flow pattern generated by a given intake shape as predicted by the simple model and by an accurate CFD model. Finally, the off-design prediction of the air intake produced by the simplified model will be used to estimate the overall fuel mass fraction requirement of a vehicle flying along a constant dynamic pressure flight corridor. The main conclusion of this analysis is that the intake geometry, which is optimal over a flight corridor, differs from that optimal at the design point, and the reasons causing this circumstance can be clearly identified.

Reference Intake Geometry

The hypersonic flight requirement of a deep integration between the vehicle aerodynamic structures and its propulsion system rules out high drag axisymmetric intake configurations in favour of two-dimensional mixed-compression intakes with forebody precompression. Most of the recent hypersonic vehicle concepts, such as the Hyper-X\textsuperscript{7,8} demonstrator, adopt this configuration which involves a shape design with several external and internal compression ramps.

The reference intake geometry for our approximate model should therefore be able on one hand to reproduce the main flow features of a two-dimensional mixed-compression intake and on the other hand to display an off-design shock pattern involving a limited number of shock-shock and shock-wall interactions. In fact, the difficulty in dealing with complex phenomena such as multiple shock interactions and reflections\textsuperscript{5} without invoking CFD analysis tools, will force us to consider very simple intake configurations only.

For this reason, the subcritical, off-design, shock pattern generated by the interaction between the bow-shock and the external ramp oblique shocks should be kept at a minimum level of complexity thereby ruling out multi-ramp configurations in spite of their higher on-design efficiencies. A single ramp geometry was therefore chosen to accomplish external compression.\textsuperscript{5}

Similar difficulties arise with multiple internal ramps when attempting to predict the successive shock reflections and interactions arising under off-design operations. Therefore, the internal compression will be accomplished by a Prandtl-Meyer single family isentropic compression profile in spite of its greater length.

![Fig. 1 Reference intake geometry and definition of the three design parameters ($M_D$, $\theta_D$, $\sigma_{th}$).](image)

Such reference intake geometry can be uniquely identified by specifying the following three independent design parameters (Fig. 1): (i) the intake design Mach number $M_D$, (ii) the oblique shock wave angle $\theta_D$ at the design Mach number and (iii) the flow deviation angle $\sigma_{th}$ downstream of the throat.

Note that the intake design Mach number $M_D$ is lower than the free stream Mach number $M_\infty$, since the flow is decelerated across the oblique shocks generated by the ramps located along the vehicle forebody.

Clearly, the wedge angle $\delta_{ramp}$ is uniquely identified once the design Mach number and the oblique shock wave angle $\theta_D$ are given.

The on-design capture area $A_0$ is taken equal to the intake duct height $A_c$. The length of the external compression is such that the oblique shock wave hits the intake cowl lip at the design Mach number.

Thus, each pair of values $(\theta_D, \sigma_{th})$ uniquely identifies a geometry within the family of intakes having a design Mach $M_D$.

By conventionally setting the intake duct height $A_c$ equal to unity, both the external ramp length $L_N$ and the intake duct entry area $A_{entry}$ can thus be found:

$$L_N = \frac{1}{\tan \theta_D}$$

$$A_{entry} = (1 - L_N \tan \delta_{ramp}) \cos \delta_{ramp}$$

The Prandtl-Meyer compression wave realizing the internal compression is centered at the intersection of the ramp wedge and a line stemming from the cowl lip and with a slope equal to the Mach line angle evaluated downstream the leading oblique shock. The intake duct geometry is found by tracing a streamline starting at the cowl lip through the Prandtl-Meyer compression. At the end of the compression, the intake cross section area defines the intake throat $A_{th}$ where the flow Mach number is minimum $M_{th}$ and, by construction, the flow angle is $\sigma_{th}$.

The proposed intake configuration provides the simplest way to realize a mixed external-internal compression, and as such has little chances to represent a "real world" intake. Moreover, no viscous effects will be accounted for in predicting the performance delivered, and the performance estimate will be overly optimistic.
since no wave losses are associated to the Prandtl-Meyer compression, whereas oblique shocks (at least one) always participate to the internal compression in a real intake. However, this simplified configuration will suffice to highlight the main phenomena affecting the choice and identification of an intake design optimal over a range of flight Mach number.

**On-Design Performance Analysis**

For a given design Mach number $M_D$, the intake geometry design parameters ($\theta_D, \sigma_{th}$) will be subject to geometrical and gasodynamical constraints, which will define the domain of feasible intake geometries. For a given shock angle $\theta_D \in (\theta_{D_{\text{min}}, \theta_{D_{\text{max}}}})$, with $\theta_{D_{\text{min}}}$ equal to the flow Mach angle (a function of $M_D$) and $\theta_{D_{\text{max}}}$ equal to the detachment angle (a function of $M_D$ and $\delta_{\text{ramp}}$), the deviation angle $\sigma_{th}$ will lie between two extremes, one corresponding to a limiting isentropic compression yielding sonic throat conditions and the other corresponding to a zero strength compression, so that $\sigma_{th} = \delta_{\text{ramp}}(M_D, \theta_D)$.

Any point chosen within the feasible domain corresponds to a physically admissible air intake geometry.

Appropriate performance indices at on-design operation can be computed for such a geometry, these being (i) adiabatic compression efficiency $\eta_c$, defined as the ratio of the isentropic enthalpy change over the actual enthalpy change accomplished through the external oblique shock and a normal shock at the throat, and (ii) the engine specific impulse $I_{sp}$, or equivalently, the specific thrust $T_{sp}$. The latter index is computed by carrying out a First Law analysis of the engine cycle with prescribed combustion and expansion efficiencies as illustrated in the Appendix.

At this stage, performance maps of adiabatic compression efficiency and specific impulse can be constructed as a function of $\theta_D$ and $\sigma_{th}$ for a given design Mach number $M_D$. Fig. 2 shows one of such maps for the design Mach number $M_D = 5.0$ intake family.

Within each feasible two-dimensional domain in the $(\theta_D, \sigma_{th})$ plane, there exists a one-dimensional sub-domain of intake geometries (identified by a dashed line in Fig.2) yielding a selection of configurations which are optimal in terms of highest adiabatic efficiency $\eta_c$, that is, for a given value of $\sigma_{th}$, there exists an optimal value of $\theta_D$ such that the configuration of the oblique shock plus the normal throat shock yields a minimum total pressure loss. The peak performance is found at the left side of this one-dimensional sub-domain and suggests that low oblique shock angles intake geometries defined for the design Mach number $M_D = 5.0$, by $\theta_D \approx 12\,\text{deg}$, with a very strong flow turning provided by the Prandtl-Meyer compression $\sigma_{th} \approx -70\,\text{deg}$ yield the maximum performance of $\eta_c \approx 0.99$ and $I_{sp} \approx 4600$. Clearly, this class of geometries is unacceptable because they are too long, the flow turning too extreme and the predicted performance over-optimistic, since no viscous effects are accounted for, and no wave losses are associated to the Prandtl-Meyer compression.

**An Approximate Shock Pattern Model**

In assessing the conclusions drawn from the inspection of the on-design performance maps, one has to consider that an optimal on-design geometry will, most likely, no longer be optimal under off-design operations, and especially so in the subcritical regime. From this follows the importance of devising a subcritical shock pattern prediction model capable of computing ‘real time’ performance parameters along a given constant dynamic pressure trajectory. Such a model should require very little CPU time, so that it could be incorporated in an automatic optimization procedure.

The model presented in this work is an extension of the model originally developed and illustrated by Seddon and Goldsmith (1985).3

Given an intake geometry and flight conditions, the model assumes that, during sub-critical operations, a curved bowshock stabilizes in front of the entry section thus prompting spillage of air over the cowl lip at

![Fig. 2 On-design performance maps for intakes with design Mach number $M_D = 5.0$.](image-url)
subsonic speed. The shock pattern is further complicated because of the ‘triple point’ interaction occurring between the curved bowshock and the oblique shock emerging from the ramp. An oblique shock impinging on the intake ramp, and a contact discontinuity emerge past the triple point interaction (see Fig. 3).

![Diagram of an algebraic model of the intake supercritical flow pattern.](image)

**Fig. 3** An algebraic model of the intake subcritical flow pattern.

A different, and somewhat simpler, model is required when spilling is accomplished at supersonic speed, when the oblique shock stemming from the external ramp has a slope angle larger than the value attained at the design point.

**Main Assumptions**

With reference to the sketch and the nomenclature shown in Fig. 3, the main assumptions of the model are the following:

1. The curved bowshock is assumed to describe a hyperbolic curve with vertex $x_0$ in a $x,y$ frame of reference, where the $x$-axis is parallel to the free-stream direction. The choice of a hyperbolic form is physically justified by the fact that the shock should have zero strength at infinity, and therefore one asymptote of the hyperbola will be set parallel to a Mach line of slope $\mu = \arcsin \frac{1}{M_0}$ where $M_0$ is the (current) Mach.

2. At the triple point the hyperbola has a local slope $\beta_\infty$ with respect to the horizontal; the slope $\beta_\infty$ is found by solving a two-dimensional Riemann problem generated by the interaction of the curved bowshock and the oblique shock stemming at the base of the external ramp.

3. The flow past the curved bowshock becomes subsonic and then reaccelerates to supersonic speeds when passing over the cowl lip. The sonic line intersects the curved bowshock at a point $S$. There the slope of the hyperbola $\beta_S$ equals the oblique shock detachment angle $\delta_{max}$ relative to the current Mach $M_0$. The local flow angle will therefore be $\lambda_S = \delta_{max}$. This conclusion is justified by the consideration that point $S$ must act as a dividing point between the part of bowshock downstream of which the flow is subsonic, and supersonic respectively. Therefore, the oblique shock is of the strong type underneath point $S$ and of the weak type above it. The maximum slope of a weak shock or the minimum slope of a strong shock will therefore occur where $\lambda_S = \delta_{max}$.

4. The sonic line joining the sonic point and the cowl lip is considered to be straight. This assumption is somewhat oversimplified but it is nearly impossible to account for the dependence of the actual sonic line shape with the Mach number and the intake geometry and to keep the model simple as required. Therefore, assuming the sonic line straightly will imply a systematic over/underestimation of the massflow crossing through it. This issue will require calibration of the model against comparison with CFD results.

**Solving the flow model problem**

Once these assumptions are laid out, it is necessary to find a "solution" for this simplified flow model, which involves finding the correct shock shape and location, the capture area, solving the triple point interaction, and assessing what type of shock reflection occurs at the ramp for the shock emerging out the triple point interaction, all of this for a given intake geometry and flight conditions.

A first guess of the capture area $A_0$ is initially set and fed as input to determine the bowshock’s hyperbola vertex $x_0$ which ultimately defines the bowshock’s standoff distance and hence the shock pattern. Finding $x_0$ involves an iterative procedure (Fig. 4) aimed at enforcing a continuity condition between the massflow $\dot{m}_{in}$ passing through the capture area at the upstream flow conditions, and the massflow $\dot{m}_{out}$ flowing through the sonic line extending between the intake cowl and the bowshock at the sonic flow conditions, as illustrated in the inset of Fig.3. At this point, the procedure must verify if the guessed value of capture area $A_0$, with the resulting shock pattern, produces sonic conditions at the throat. If this is not so the whole procedure is re-iterated until convergence is reached at which point the shock system found is compatible with the intake geometry and flight conditions.

The model here presented extends the original Seddon-Goldsmith model, by allowing the triple point to fall both within the capture tube (sketch at the bottom of Fig 5) and outside of it (sketch at the top of Fig 5). The first instance is more complicated than the latter because the evaluation of the capture area cannot be carried out independently of the bowshock
Fig. 4 Flow chart of the iterative procedure to find the model solution.

standoff distance estimation as the average pressure losses for the captured streamtube are functions of the standoff distance itself. The capture area relative to the latter case, on the other hand, can be determined independently of the triple point, since the captured streamtube does not embrace the triple point and therefore the related pressure losses are solely due to the oblique ramp shock and the downstream wall-impinging shock.

Allowing for the triple point to fall both outside and inside of the capture streamtube makes it possible to further estimate the flight conditions at which it actually crosses the dividing streamline (i.e. the streamline dividing the captured streamtube and the outer flow). This can be adopted as a criterion to predict the onset of buzzing oscillations (Ferri’s criterion\(^4\)).

**Generalized Exact Riemann Solver**

The Riemann problem arising at the triple point involves the interaction of a weak shock (the oblique wedge shock), and a locally strong shock (the curved subcritical shock). Such combination of shocks admits, among the emerging waves, a strong outer oblique shock, so that the flow past it is subsonic and an inner wall-impinging shock. This requires the extension of a standard exact Riemann solver to account for strong shocks. The Riemann solver yielded the results reported in Fig.6, where for a given wedge angle \(\delta_{\text{ramp}}\) and free-stream Mach \(M_0\) the local slopes \(\beta_\infty\) of the outer portion of the bowshock and \(\beta_2\) of the inner portion (see upper inset of Fig.3) are displayed.

Fig. 5 Model scheme for triple point inside and outside of capture tube.

Fig. 6 Riemann solver results: \((\beta_2, \beta_\infty)_{M_0, \delta_{\text{ramp}}}\).

The oblique shock emerging from the triple point and impinging the ramp can reflect at the wall following three distinct modalities: (i) regular oblique shock reflection, (ii) a Mach reflection or (iii) no reflection at all. In this latter case the oblique shock is strong and bends so as to impinge the ramp wall at a right
angle. Fig. 8 displays the domains of existence of the three reflection modes as a function of the ramp angle $\delta_{\text{ramp}}$ and flight Mach $M_0$.

A CFD validation procedure has confirmed that intake geometries featuring the Mach reflection mode during subcritical operation provide very unfavourable flow patterns as illustrated in Fig. 7. These are originated by the very strong velocity gradients across the vortex sheet emanating from the triple point which ultimately generate large recirculation regions within the intake’s convergent duct leading, among other things, to a drastic reduction of the captured mass-flow.

In contrast, intakes displaying no shock reflection at the ramp wall are preferable as the flowfields they originate are far more uniform and recirculation regions far smaller.

Model Validation

Once the procedure has converged, the proposed model problem delivers the following information: the capture area, all main flow states (assumed piecewise constant within each flow portion enclosed by shocks or wall boundaries), shocks’ locations, structures, and strengths. Derived quantities will include overall static temperature rise and total pressure drop across the intake.

The model was subject to an extensive validation procedure through comparison with inviscid CFD results. The CFD solver used is based on a shock-fitting technique, which allows to sharply measure shock
angles and location so as to improve the accuracy of the comparison. For those intakes which display a normal incident bowshock in subcritical operation, the comparison campaign has allowed for a thorough verification of the simplifying assumptions made and for error estimation. In particular the straight sonic line assumption was extensively investigated and the model calibrated accordingly.

Fig. 9 shows a direct comparison between the CFD generated flowfield and the algebraic model for a \( M_D = 3.5 \quad \theta_D = 25\text{deg} \quad \sigma_{th} = -10\text{deg} \) intake at flight Mach 2. The comparison shows that the triple point location is well captured although the bowshock shape is somewhat in disagreement as the shock moves into the farfield.

For the same intake geometry, Fig.10 shows the trajectory followed by the triple point as the Mach is lowered in sub-critical operation as computed by CFD and by the algebraic model. Note that the curved ascending-descending path of the triple point obtained through CFD can be properly described by the simplified algebraic model.

The algebraic model also allows us to predict the Mach number (transition Mach) below which air is spilled subsonically and above which supersonically. Fig.11 shows that for a given design Mach and oblique shock angle, the range of supersonic spillage, defined by the difference between the design Mach and the transition Mach, decreases for higher absolute values of the flow deviation, i.e. for stronger Prandtl-Meyer internal compressions. This is because for stronger on-design internal compressions the design throat Mach approaches unity thus leaving less margin before choking conditions arise as the flight Mach is lowered.

Off-Design Performance Analysis

The approximate algebraic model proposed will enable us to carry out an extensive off-design performance campaign relative to a great number of air intake shapes, along given flight path trajectories. We chose to examine intake geometries for the design Mach number \( M_D = 3.5 \), and different combinations of \( (\theta_D, \sigma_{th}) \) as detailed in Table 1.

As a global performance parameter we selected the fuel consumption needed to accelerate a reference vehicle from an initial Mach, \( M_i = 1.6 \), to a final Mach, \( M_f \) equal to the intake geometry design Mach number \( M_D = 3.5 \).

The vehicle flies along a constant pitch and constant dynamic pressure \( q_0 = 72.5kN/m^2 \) acceleration trajectory. The vehicle is assumed to have variable,
A lumped-mass body subject to weight, thrust $F$ and spillage drag $D_e$. The spillage drag is solely due to the intake and can be easily evaluated on the basis of the model output data as illustrated in the Appendix. No other drag sources have been accounted for, so as to highlight the effects of the intake off-design operations on the global performance of the vehicle.

Manipulation of the equation of motion yields a relation for the instantaneous vehicle mass:

$$\frac{dm}{m} = -\left(\frac{d\left(\frac{v^2}{r}\right)}{\eta_0 h_{PR}(1 - \frac{D_e}{F})}\right)$$

(1)

where $\eta_0$ is the total airbreathing engine efficiency as calculated through a simple First Law analysis (see Appendix) of the thermodynamic engine cycle, $r$ is the radial distance from the center of the Earth, and $h_{PR}$ is the propellant heat of reaction. The numerical integration of Eq.(1) yields the end-of-mission Fuel-to-Mass Fraction (FMF) index $\Pi_f$ defined as:

$$\Pi_f = \frac{m_i - m_f}{m_i}$$

(2)

We will define an intake geometry optimal if it will minimize the final FMF over an accelerating flight path. Note that Eq.(1) and hence $\Pi_f$ ultimately depends upon the following group of terms:

$$\eta_0 \left(1 - \frac{D_e}{F}\right)$$

(3)

which can be identifiable as an effective overall efficiency of the engine. It is therefore clear how, everything being equal, in order to save fuel it is essential to adopt an intake design that during the mission, and especially during its off-design operation, minimizes the drag to thrust ratio $D_e/F$.

Fig. 13 Effective overall efficiency along trajectory as a function of Mach for several intake design parameters $\sigma_{th}$ and fixed $\theta_D = 20\text{deg}$.

Fig. 14 Instantaneous Fuel Mass Fraction along trajectory as a function of Mach for several intake design parameters $\sigma_{th}$ and fixed $\theta_D = 20\text{deg}$.

Fig. 15 End-of-mission Fuel Mass Fraction as a function of intake design parameters.
The importance of intake spillage drag appears clearly whenever it is neglected in the computation of the final FMF. We observe the emergence of optimal values for $\sigma_{th}$ for given values of $\theta_{p}$ which minimize the final FMF. Considering intake geometries with increasing absolute values of $|\sigma_{th}|$, from right to left in the figure, we initially observe that an increase in $\eta_{th}$ has a beneficial effect on FMF (in spite of increasingly wider subsonic spillage ranges) until we reach an optimum beyond which the deleterious effect of increasing $D_{c}/F$ (and yet wider subsonic spillage ranges) prevails.

This result clearly underlines that optimal, fixed-geometry, intake design geometries exist for mission profiles involving a continuous acceleration flight path and such geometries are different from those found optimal at constant Mach flight path.

**Conclusions**

A simplified model for the subcritical behaviour of a supersonic fixed-geometry, mixed-compression air intake was presented. The algebraic nature of the model was validated against detailed CFD calculations of the flowfield about the intake. For geometries which do not manifest shock reflection at the ramp, and hence have widely uniform flowfields, the model delivers a satisfactory agreement with CFD data.

The model was successful in pinpointing the main mechanisms governing the relationships between intake geometry, subcritical shock patterns and ultimately compression efficiency.

Given the low computing power requirements, it was possible to carry out an extensive off-design engine performance analysis so that the parametrical influence of intake design parameters could be clearly highlighted. Optimal geometries were found for which fuel consumption is minimal over a given accelerating trajectory. In particular it was found that such optimal geometries arise because intakes delivering the best performance both at design point and during supersonic spillage, simultaneously show the highest drag/thrust ratios during the subsonic spillage regime and hence the worst performance in such regime. A compromise geometry will therefore sacrifice part of its on-design efficiency in favour of lower drag values during the subsonic spillage regime.

The subsonic spillage, or unstarted, operation is clearly a highly unfavourable flight condition and certainly one to be avoided during cruise and acceleration. To pursue this aim a well established solution is the internal bleeding of part of the captured air flow which causes the bowshock to be swallowed and started conditions to be reached. The proposed model can indeed yield a proper estimate of the amount of bleeding required. Since its main outcome is a relationship between the bowshock standoff, the captured mass flow and throat area at given flight conditions, then the amount of bleeding is simply found by enforcing that the standoff distance be zero. Under this limiting condition, the bowshock, at the inlet duct, becomes unstable, is swallowed and the intake is started. Analysis of the bleeding requirements to achieve supersonic spillage operation along the whole mission profile will be the subject of a forthcoming research activity.

**Appendix**

**First Law Analysis**

Engine modelling can be kept at a minimum level of complexity through a First Law Analysis which views the propulsion plant as a thermodynamic closed cycle. In so doing, engine global efficiency can be defined as the ratio of thrust power to input thermal power, and can be expressed (see Fig. 16) as the product of a combustion efficiency $\eta_{b}$, a propulsive efficiency $\eta_{p}$ and the thermodynamic cycle’s efficiency $\eta_{e}$:

$$\eta_{0} \equiv \frac{FV_{0}}{\dot{m}_{f}h_{PR}} = \eta_{b} \eta_{c} \eta_{p}$$

where $h_{PR}$ is the heat of reaction, $\dot{m}_{f}$ is the fuel mass flow and where $V_{0}$ is the flight velocity.

![Fig. 16 Global engine efficiency](image)

With reference to the nomenclature of Fig.17 such efficiencies are defined as

$$\eta_{b} = \frac{h_{4} - h_{3}}{f h_{PR}}$$

$$\eta_{c} = \frac{(h_{4} - h_{3}) - (h_{10} - h_{0})}{h_{4} - h_{3}}$$

$$\eta_{p} = \frac{FV_{0}}{[(h_{4} - h_{3}) - (h_{10} - h_{0})] \dot{m}_{0}} \approx \frac{FV_{0}}{[(\dot{m}_{f} + \dot{m}_{0}) \frac{h_{3}}{\eta_{c}} - \dot{m}_{0}] \frac{h_{3}}{\eta_{c}}}$$

where $\dot{m}_{0}$ is the captured air mass flow and $f = \dot{m}_{f}/\dot{m}_{0}$ is the fuel to air ratio.

Compression, combustion and expansion submodels can then be implemented in series, each representing, with its own efficiency, an energy transformation of the cycle. By assigning proper constant values to the combustion efficiency and the nozzle adiabatic expansion efficiency $\eta_{e}$, the effects on the global engine efficiency of the intake’s adiabatic compression efficiency $\eta_{c}$ can be adequately singled out.

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With reference to Fig. 17, adiabatic compression efficiency is defined as

$$\eta_c(\psi, \pi_c) = \frac{h_3 - h_1}{h_3 - h_0} = \psi - \left(\frac{1}{\pi_c}\right) \frac{\psi - 1}{\psi} (8)$$

where $\psi \equiv T_3/T_0$ is the temperature rise and $\pi_c \equiv p_3/p_0$ the total pressure drop across the intake. Both $\psi$ and $\pi_c$ are directly extracted as output of the intake model as a function of flight conditions.

Carrying out the complete cycle analysis, yields a relationship between thermodynamic cycle efficiency and adiabatic compression efficiency:

$$\eta_c = 1 - \left\{ \left(\frac{\psi - C_pT_0}{\eta b f h_{PR}} + 1\right) \frac{T_{10}}{T_4} - \frac{C_pT_0}{\eta b f h_{PR}} \right\} (9)$$

where

$$\frac{T_{10}}{T_4} = 1 - \eta_c \left\{ 1 - \left[ 1 - \left( 1 - \frac{1}{\psi_c} \right) \right] \right\} (10)$$

with $\eta_c \equiv (h_4 - h_{10})/(h_4 - h_\gamma)$ being the adiabatic expansion efficiency.

![Fig. 17 Thermodynamic cycle nomenclature](image)

Once the engine global efficiency is computed, the uninstalled thrust can be computed as $F = \eta_{0i} m_f h_{PR}/V_0$ and specific impulse as $I_{sp} \equiv F/(m_f g_0) = \eta_h h_{PR}/(g_0 V_0)$.

**Spillage Drag**

A common definition of intake drag, consistent with the definition of uninstalled thrust, generally includes a cowl drag and a pre-entry drag contribution. The former includes viscous and suction effects on the cowl while the latter accounts for the resulting pressure forces on the captured streamtube. Spillage drag, on the other hand, is defined as the drag increase as the capture area is lowered below full-flow conditions. For supersonic intakes in subcritical operation, neglecting cowl suction (thin wall assumption) and viscous effects, spillage drag can be assumed equal to pre-entry drag. An expression for supersonic subcritical spillage drag can be derived by extending the Fraenkel\(^1\) (1950) expression valid for a Pitot intake to single-wedge intake.

For the supersonic spillage case (see Fig. 18.1) the following expression was used:

$$D_c = (p_1 - p_0) (A_c - A_0) (11)$$

Fig. 18 Spillage Drag

where $A_0$ is the capture area, $A_c$ the intake entry area, $p_0$ and $p_1$ the pre- and post-shock static pressures.

For the subsonic spillage regime, the case in which the dividing streamline is below the triple point (see Fig. 18.2a) yields the following expression:

$$D_c = (p_1 - p_0) (A_c - A_0) + \frac{(p_{a2} - p_0) (A_c - A_1)}{} (12)$$

whereas the case in which the dividing streamline is above the triple point (Fig. 18.2b) yields

$$D_c = (p_{a3} - p_0) (A_c - A_0). (13)$$

**References**

6. Czysz P., Bruno C., Kanerori K., "Interaction of the Propulsion System and System Parameters Determines the Design Space Available for Solution". Copyright 2001 by Paul A. Czysz. Published by the American Institute of Aeronautics and Astronautics, Inc. with permission. Released to the AIAA to publish in all forms.


